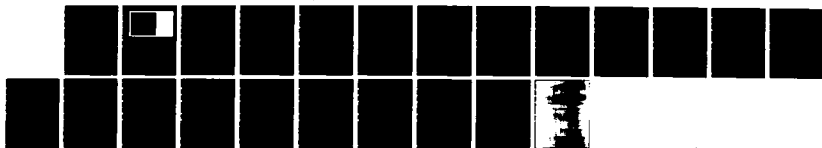
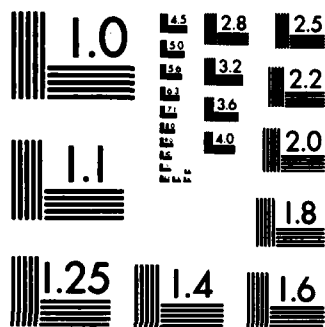


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MRC Technical Summary Report #0112

NUMERICAL SOLUTIONS
FOR CAVITATING FLOW OF A FLUID
WITH SURFACE TENSION
PAST A CURVED OBSTACLE

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ABSTRACT

↓
The problem of cavitating flow past a two-dimensional curved obstacle is considered. Surface tension is included in the dynamic boundary condition. The problem is solved numerically by series truncation. Explicit solutions are presented for the flow past a circle. It is shown that for each value of the surface tension different from zero, there exists a unique solution which leaves the obstacle tangentially. As the surface tension approaches zero, this solution tends to the classical solution satisfying the Brillouin - Villat condition. Vanden-Broeck considered the effect of surface tension on the cavitating flow past a flat plate and on the shape of a jet emerging from a reservoir. His results indicate that the velocity is infinite at the separation points. It is shown that these unbounded values of the velocity are removed when the thickness and finite curvature of the ends of the plate and of the ends of the walls of the reservoir are taken into account. 4

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The results presented here should be relevant to any physical situation in which the free surface of a fluid intersects a solid surface.

The responsibility for the wording and views expressed in this descriptive summary lies with MRC, and not with the author of this report.

NUMERICAL SOLUTIONS FOR CAVITATING FLOW
OF A FLUID WITH SURFACE TENSION PAST A CURVED OBSTACLE

Jean-Marc Vanden-Broeck

1. Introduction

We consider the cavitating flow past a two-dimensional curved obstacle (see Fig. 1). We neglect the effects of viscosity and compressibility. The cavity is characterized by its constant pressure p_c and its surface tension T while the fluid has density ρ , pressure p_c at infinity and constant velocity U at infinity. We restrict our attention to obstacles which are symmetrical with respect to the direction of velocity at infinity.

The problem with $T = 0$ has been considered by many previous investigators (see Birkhoff and Zarantonello [1] for a review). Efficient numerical schemes were derived by Brodetsky [2], Vanden-Broeck [3] and others. The results of these calculations show that a solution exists for all positions of the separation points A and B . This degeneracy is usually removed by imposing the Brillouin - Villat condition, which requires the curvature of the free surface to be finite at the separation points A and B .

The effect of surface tension on the cavitating flow past a curved obstacle was considered by Vanden-Broeck [3]. Vanden-Broeck derived an asymptotic solution for T small. He found that for most positions of the separation points, the flow does not leave the obstacle tangentially. In addition he showed the existence for each value of T of a particular position of the separation points A and B for which the slope is continuous at A and B . However he did not compute these solutions explicitly.

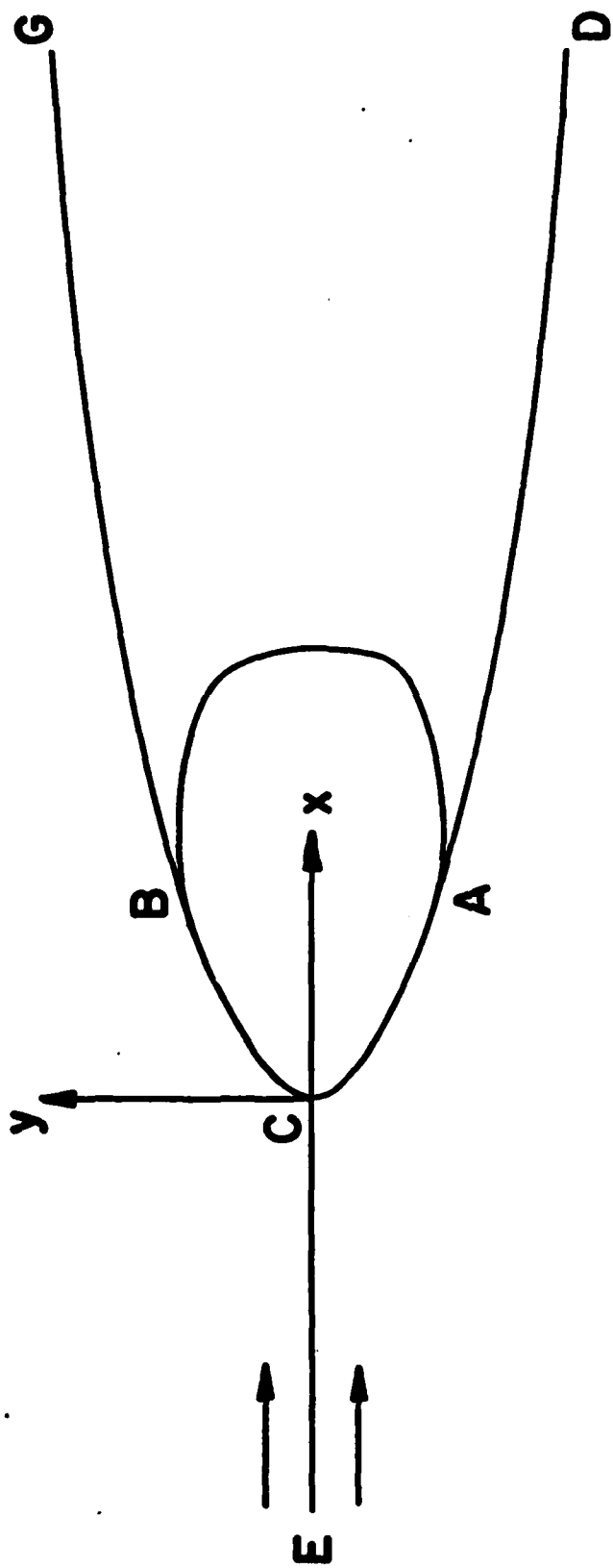


Figure 1: Sketch of the flow and the coordinates.

In the present paper we solve the cavitating flow problem numerically by series truncation. We assume that the flow leaves the obstacle tangentially. Explicit solutions are presented for the flow past a circle. Our results confirm Vanden-Broeck's [3] findings. For each value of T there exists a unique solution which leaves the curved obstacle tangentially. As $T \rightarrow 0$, these solutions approach the classical solution satisfying the Brillouin - Villat condition. Therefore the degeneracy of the problem with $T = 0$ is removed by solving the problem with $T \neq 0$ and then taking the limit as $T \rightarrow 0$.

Vanden-Broeck [4] investigated the effect of surface tension on the cavitating flow past a flat plate. He found that the velocity at the separation points is infinite for all values of $T \neq 0$. We show that these unbounded values of the velocity are removed by replacing the plate by a thin ellipse.

The problem is formulated in Section 2. The numerical scheme is presented in Section 3 and the results are discussed in Section 4.

2. Formulation

Let us consider the cavitating flow past a curved smooth and symmetrical obstacle. We choose Cartesian coordinates such that the flow is symmetrical with respect to the x -axis (see Fig. 1). We denote by L a typical length of the obstacle and by U the velocity at infinity. We choose L as the unit length and U as the unit velocity.

We introduce the potential function $b\phi$ and the stream function $b\psi$. The constant b is chosen such that $\phi = 1$ at the separation points A and B . Furthermore we choose $\phi = 0$ at $x = y = 0$ and $\psi = 0$ on the free surface. The flow configuration in the complex potential plane $f = \phi + i\psi$ is illustrated in Fig. 2.

We denote the complex velocity by $\zeta = u - iv$ and we define the function $\tau - i\theta$ by the relation

$$\zeta = u - iv = \frac{1}{x_\phi + iy_\phi} = e^{\tau - i\theta} . \quad (1)$$

We shall seek $\tau - i\theta$ as an analytic of $f = \phi + i\psi$ in the half plane $\psi < 0$.

On the surface of the cavity the Bernoulli equation and the pressure jump due to surface tension yield (see Ackerberg [5] for details)

$$\frac{e^\tau}{b} \frac{\partial \theta}{\partial \phi} = \frac{1}{2} \alpha (e^{2\tau} - 1) \quad (2)$$

$$1 < \phi < \infty .$$

Here α is the Weber number defined by

$$\alpha = \frac{\rho U^2 L}{\tau} . \quad (3)$$

We shall seek solutions for which the flow leaves the obstacle tangentially. Therefore we require the velocity to be finite at $\phi = 1$. Relation (2) shows that the curvature of the free surface $\frac{e^\tau}{b} \frac{\partial \theta}{\partial \phi}$ is then also finite at $\phi = 1$ for $\alpha \neq \infty$. Thus we impose the condition

$$\left| \frac{\partial \theta}{\partial \phi} \right| < \infty \text{ at } \phi = 1 .$$

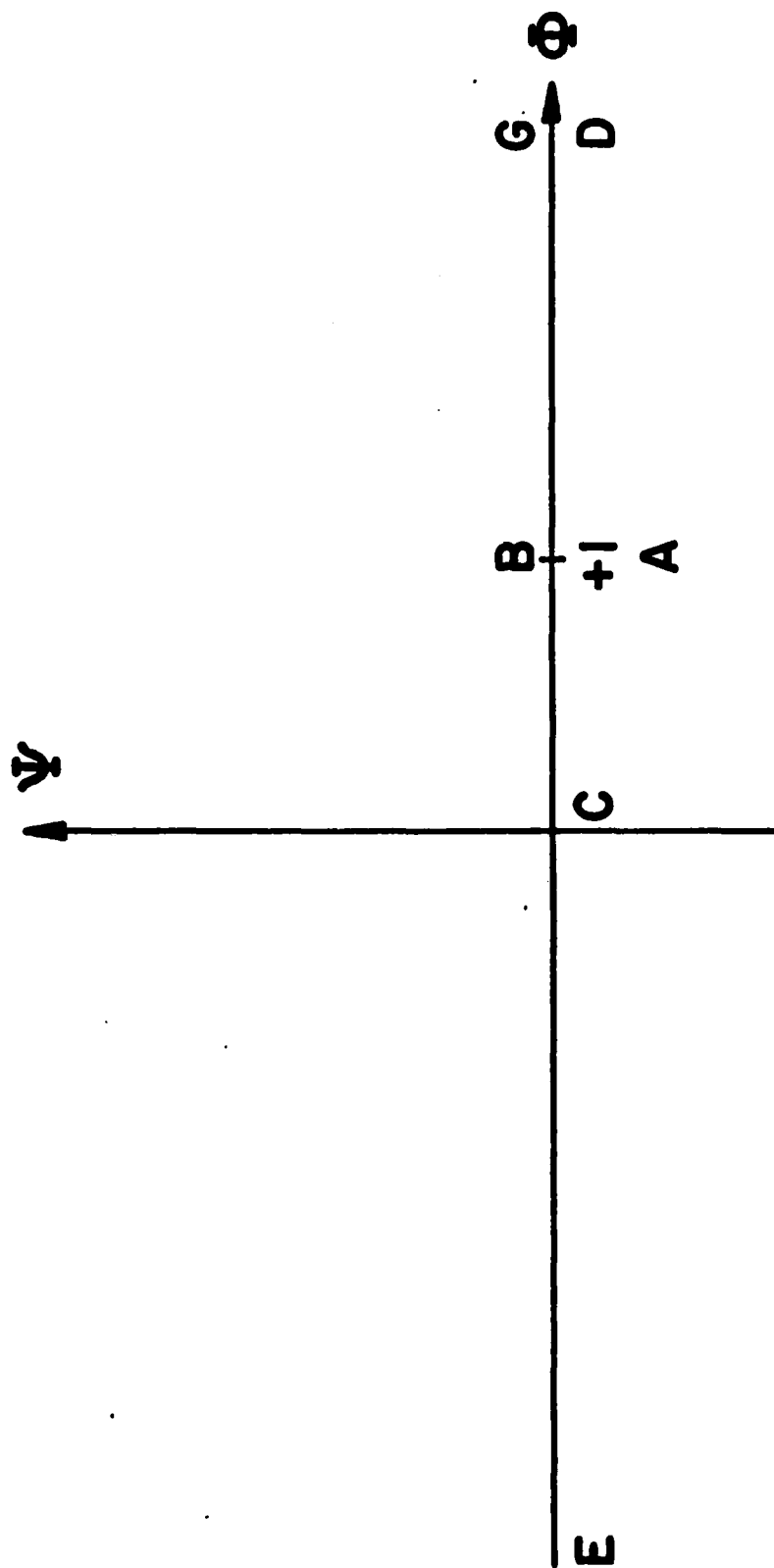


Figure 2: The image of the flow in the plane of the complex potential

$$f = \phi + i\psi.$$

However we do not require the curvature of the free surface to be equal to the curvature of the obstacle. Therefore we allow a finite jump discontinuity of $\frac{\partial \theta}{\partial \phi}$ at $\phi = 1$.

Finally the symmetry of the problem and the kinematic condition on the obstacle yield

$$\theta(\phi) = 0 \quad \psi = 0, \phi < 0 \quad (4)$$

$$F[x(\phi), y(\phi)] = 0 \quad \psi = 0, 0 < \phi < 1 \quad (5)$$

Here $F(x, y) = 0$ is the equation of the shape of the obstacle and $\theta(\phi)$, $x(\phi)$ and $y(\phi)$ denote respectively $\theta(\phi, 0_-)$, $x(\phi, 0_-)$ and $y(\phi, 0_-)$.

This completes the formulation of the problem of determining the function $\tau - i\theta$. For each value of α , $\tau - i\theta$ must be analytic in $\psi < 0$ and satisfy the boundary conditions (2), (4) and (5).

3. Numerical procedure

We define the new variable t by the transformation

$$f = i \frac{t+1}{t-1} . \quad (6)$$

This transformation maps the lower half plane $\psi < 0$ onto the unit circle in the complex t -plane so that the free-surface, the boundary of the obstacle and the negative x -axis go onto the circumference (see Fig. 3).

For x and y small the flow can be locally described as the flow inside a right angle corner. Using (6) we obtain

$$\zeta \sim (t+1)^{1/2} \text{ as } t \rightarrow -1 . \quad (7)$$

At infinity we require the velocity to be unity in the x -direction. Hence

$$\zeta \rightarrow 1 \text{ as } t \rightarrow 1 . \quad (8)$$

We shall represent ζ by the expansion

$$\zeta = e^{\tau-i\theta} = (t+1)^{1/2} \sum_{n=0}^{\infty} (C_{n+1} + i d_{n+1}) t^n . \quad (9)$$

The expansion (9) satisfies (7). The unknown coefficients C_n and d_n and the constant b have to be determined to satisfy (2), (4), (5) and (8).

We use the notation $t = |t|e^{i\sigma}$ so that the streamline $\psi = 0$ is given by $t = e^{i\sigma}$, $0 < \sigma < 2\pi$. Using (6) we rewrite (2) in the form

$$\frac{\pi}{b} 2 \sin^2 \frac{\sigma}{2} e^{\tilde{\tau}} \frac{d\tilde{\theta}}{d\sigma} = \frac{1}{2} \alpha (e^{2\tilde{\tau}} - 1) \quad 0 < \sigma < \frac{\pi}{2} . \quad (10)$$

Here $\tilde{\tau}(\sigma)$ and $\tilde{\theta}(\sigma)$ denote the values of τ and θ on the circumference. The functions $\tilde{\tau}(\sigma)$ and $\tilde{\theta}(\sigma)$ can be evaluated in terms of C_n , d_n and b by substituting $t = e^{i\sigma}$ in (9).

In order to evaluate $\frac{d\tilde{\theta}}{d\sigma}$ for $0 < \sigma < \frac{\pi}{2}$, we differentiate term by term the expansion for $\tilde{\theta}(\sigma)$. This new expansion can be expected to converge to $\frac{d\tilde{\theta}}{d\sigma}$ for $0 < \sigma < \frac{\pi}{2}$ and to $\frac{1}{2} \left[\left(\frac{d\tilde{\theta}}{d\sigma} \right)_{\sigma=(\frac{\pi}{2})_-} + \left(\frac{d\tilde{\theta}}{d\sigma} \right)_{\sigma=(\frac{\pi}{2})_+} \right]$ for $\sigma = \frac{\pi}{2}$ where we

anticipate a discontinuity in curvature.

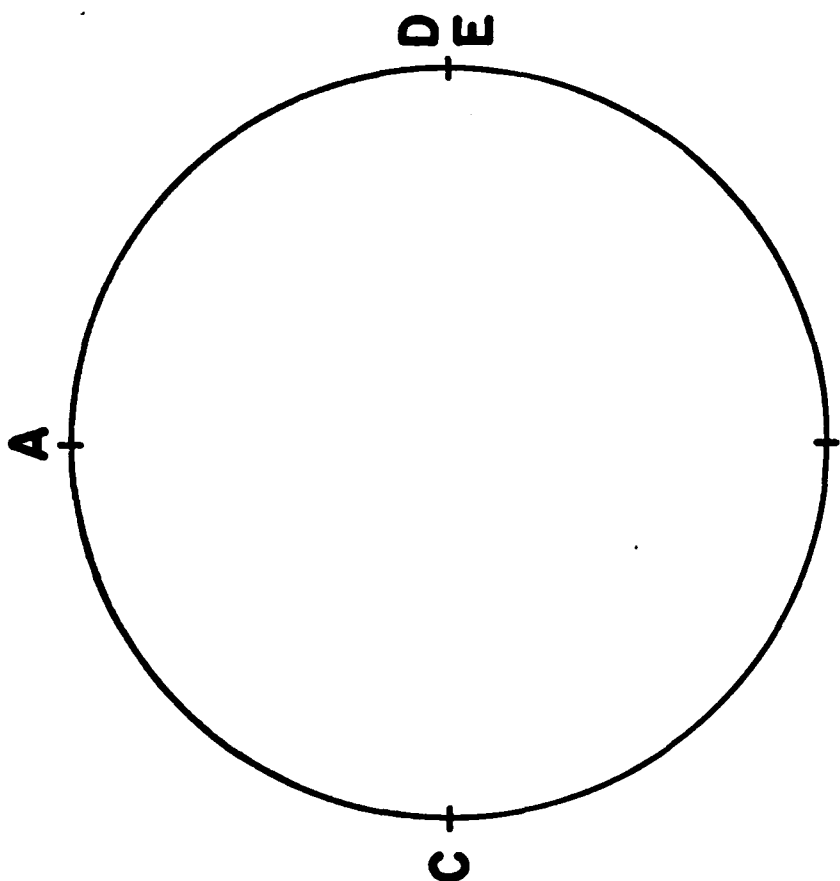


Figure 3: The t-plane.

We solve the problem approximately by truncating the infinite series in (9) after N terms. For convenience we choose N to be odd. We find the $2N$ coefficients C_n and d_n and the constant b by collocation. Thus we introduce the $2N - 2$ mesh points

$$\sigma_I = \frac{2\pi}{2N-2} \left(I - \frac{1}{2} \right), \quad I = 1, \dots, 2N-2. \quad (11)$$

Using (9) and (11) we obtain $[\tilde{\tau}(\sigma)]_{\sigma=\sigma_I}$, $[\tilde{\theta}(\sigma)]_{\sigma=\sigma_I}$ and $[\frac{d\tilde{\theta}}{d\sigma}]_{\sigma=\sigma_I}$ in terms of the coefficients C_n and d_n . Substituting these expressions into (4) and (10) we obtain $3(N-1)/2$ equations.

From (1) and (6) we obtain

$$\frac{d\tilde{x}}{d\sigma} = \frac{-b}{2 \sin^2 \frac{\sigma}{2}} e^{-\tilde{\tau}} \cos \tilde{\theta} \quad (12)$$

$$\frac{d\tilde{y}}{d\sigma} = \frac{-b}{2 \sin^2 \frac{\sigma}{2}} e^{-\tilde{\tau}} \sin \tilde{\theta}. \quad (13)$$

Relations (12) and (13) enable us to calculate $[\frac{d\tilde{x}}{d\sigma}]_{\sigma=\sigma_I}$ and $[\frac{d\tilde{y}}{d\sigma}]_{\sigma=\sigma_I}$ in terms of C_n , d_n and b . Using the trapezoidal rule we then calculate $\tilde{x}(\sigma_I)$ and $\tilde{y}(\sigma_I)$.

The boundary condition (5) can now be rewritten as

$$F[\tilde{x}(\sigma_I), \tilde{y}(\sigma_I)] = 0 \quad (14)$$

$$I = \frac{N+1}{2}, \dots, N-1$$

Relation (14) provides $\frac{N-1}{2}$ extra equations. Therefore we have $2N-2$ nonlinear equations for the $2N+1$ unknowns C_n , d_n and b . The condition (8) provides the extra equation

$$[\tilde{\tau}(\sigma)]_{\sigma=0} = 0. \quad (15)$$

The last two equations are obtained by imposing (4) at $\sigma = \frac{3\pi}{2}$ and (10) at $\sigma = \frac{\pi}{2}$. Thus we obtain for each value of α a system of $2N+1$ nonlinear algebraic equations for the $2N+1$ unknowns C_n , d_n and b . The system is

solved by Newton method. Once the system is solved, the shape of the free surface is then given parametrically by $\tilde{x}(\sigma_I), \tilde{y}(\sigma_I), I = 1, \dots, \frac{N-1}{2}$.

4. Discussion of the results

We consider the cavitating flow past a circle. We choose the reference length L as the radius of the circle. Therefore we write

$$F(x,y) = (x-1)^2 + y^2 - 1. \quad (16)$$

We used the numerical scheme of section 3 to compute solutions for various values of α . The coefficients C_n and d_n were found to decrease rapidly as n increases.

Our numerical results confirm Vanden-Broeck's findings. For each value of $\alpha \neq \infty$, there exists a unique solution that leaves the obstacle tangentially. Typical profiles of the free surface for $\alpha = 0, 2$ and ∞ are shown in Fig. 4. Each of these profiles is characterized by a different angular position γ of the separation points. Values of γ as a function of α^{-1} are shown in Fig. 5.

As α approaches infinity, our solution tends to the classical free-streamline solution satisfying the Brillouin - Villat condition. This solution is characterized by $\gamma = \gamma^* \sim 55^\circ$. Therefore the degeneracy of the problem with $T = 0$ is removed by solving the problem with $T \neq 0$ and then taking the limit as $T \rightarrow 0$.

As $\alpha \rightarrow 0$, $\gamma \rightarrow 90^\circ$ and the free surface profile approaches two horizontal lines parallel to the direction of the velocity at infinity (see Fig. 4).

Similar results are found for the cavitating flow past an ellipse. The length L is chosen as the half-length of the axis parallel to the y -axis and the ellipses are parametrized by the dimensionless half-length r of the axis parallel to the x -axis (see Fig. 6). The previous results for the flow past a circle corresponds to $r = 1$.

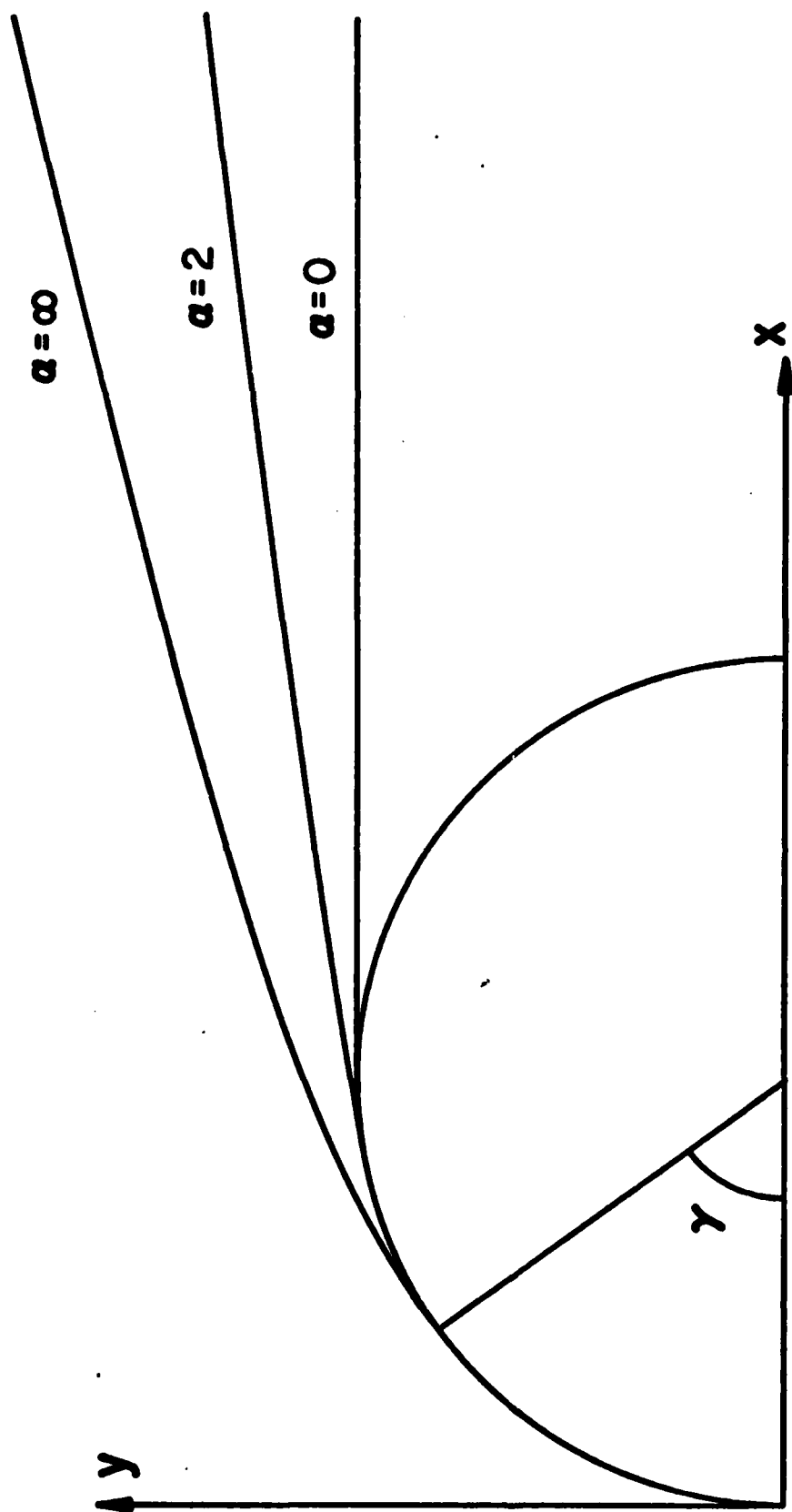


Figure 4: Cavities in steady two-dimensional flow past a circle for various values of the Weber number α .

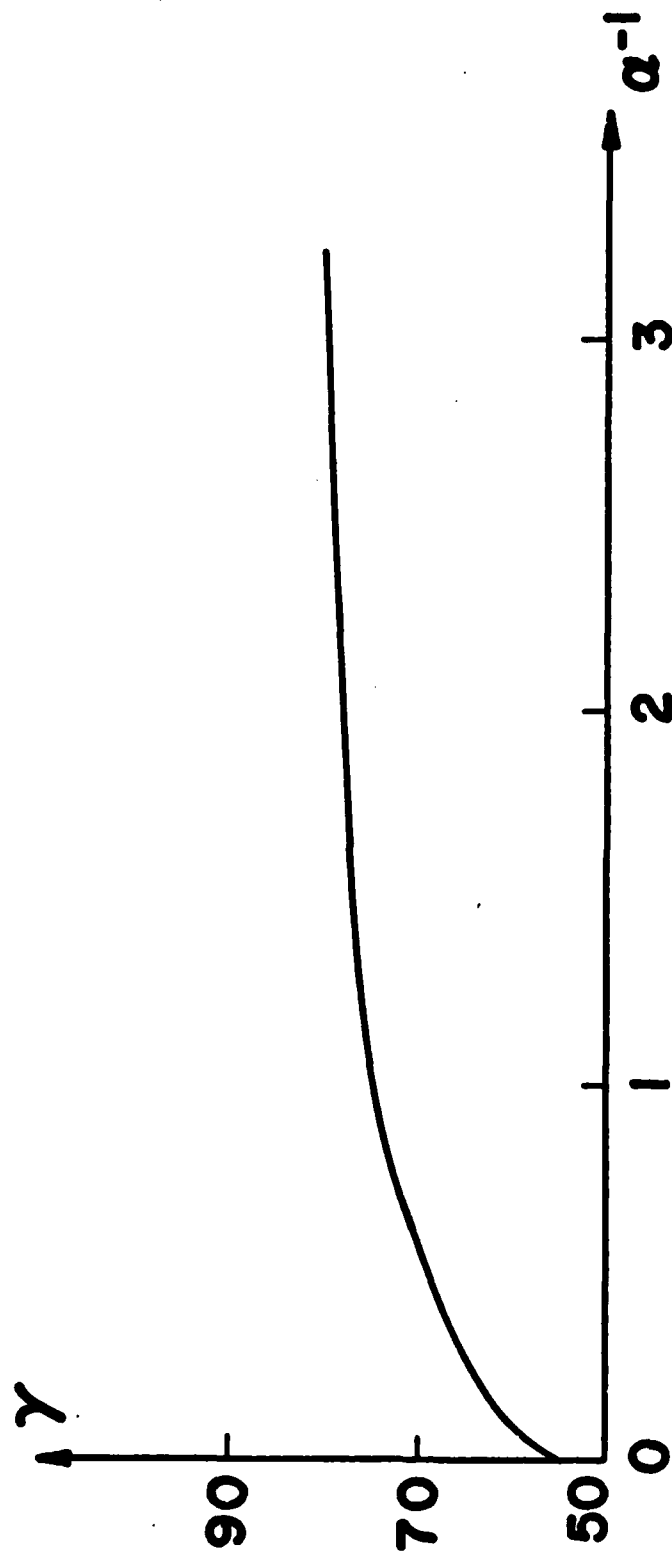


Figure 5: Values of the angular position γ of the separation points for the cavitating flow past a circle as a function of α^{-1} .

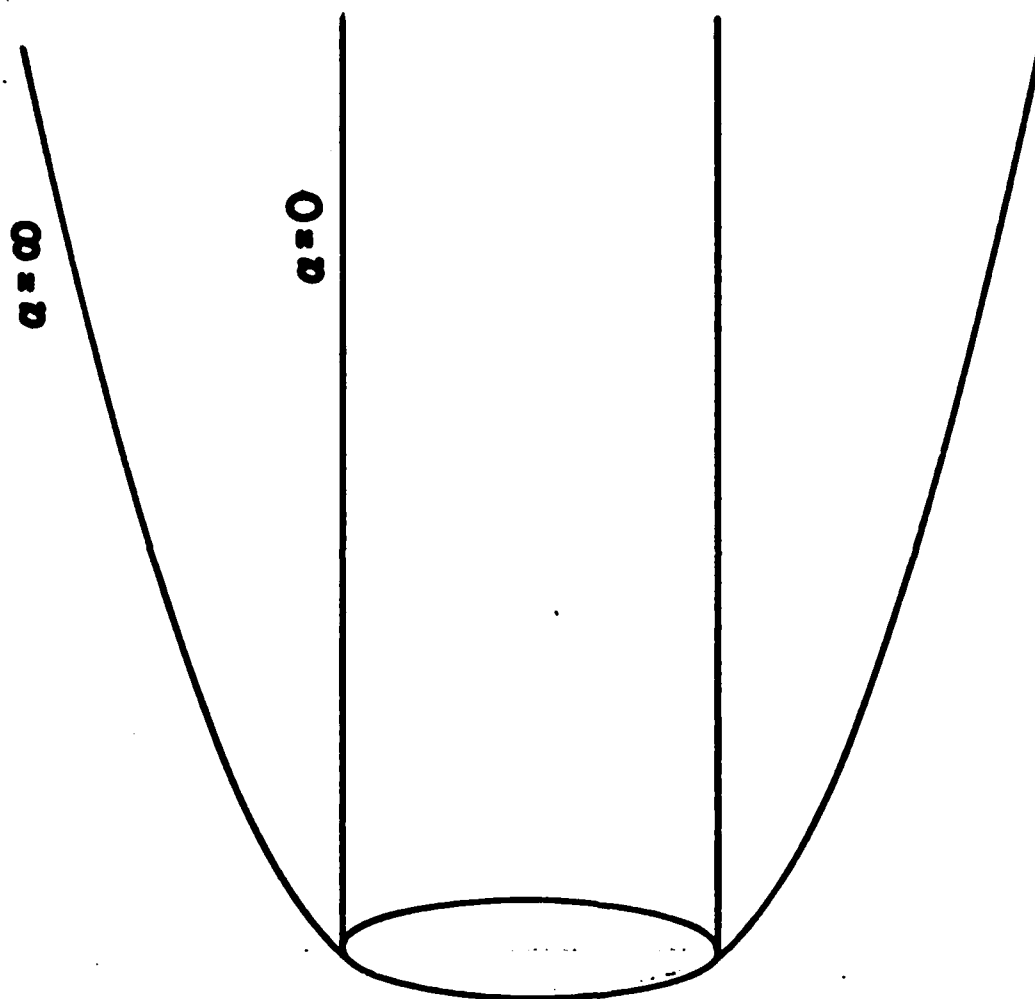


Figure 6: Cavities in steady two-dimensional flow past an ellipse for $\alpha = 0$ and $\alpha = \infty$. The ratio of the length of the axis of the ellipse is 0.2.

The limiting configurations $\alpha \rightarrow 0$ and $\alpha \rightarrow \infty$ for $r = 0.2$ are shown in Fig. 6. The solution obtained in the limit $\alpha \rightarrow \infty$ is the classical solution satisfying the Brillouin - Villat condition. The solutions corresponding to the limit $\alpha \rightarrow 0$ are characterized by a separation point at $x = r$ and $y = 1$.

As $r \rightarrow 0$, the ellipse approaches a flat plate parallel to the y-axis and the separation point of the limiting solution $\alpha \rightarrow \infty$ approaches the separation point $x = r, y = 1$ of the limiting solution $\alpha \rightarrow 0$. Therefore all the solutions for r small and $0 < \alpha < \infty$ have their separation points in the immediate neighbourhood of $x = r, y = 0$.

Vanden-Broeck [4] considered the effect of surface tension on the cavitating flow past a flat plate (i.e., the limiting problem $r = 0$). He provided analytical and numerical evidence that the velocity at the separation points is infinite for all values of $\alpha \neq \infty$. Our results show that these unbounded values of the velocity are removed by replacing the flat plate by a thin ellipse (i.e., by solving the problem with $r \neq 0$ and small).

Vanden-Broeck [6] investigated the effect of surface tension on the shape of a jet emerging from a reservoir. He also found unbounded values of the velocity at the separation points. Our results indicate that these unbounded values of the velocity can be removed by taking into account the finite thickness and finite curvature of the ends of the walls of the reservoir.

REFERENCES

1. G. Birkhoff and E. Zarantonello, *Jets, Wakes and Cavities*, Academic Press, New York (1957).
2. S. Brodetsky, *Proc. R. Soc. Lond.* A102, 542 (1923).
3. J.-M. Vanden-Broeck, *J. Fluid Mech.* 133, 255 (1983).
4. J.-M. Vanden-Broeck, *Q. J. Mech. Appl. Maths.* 34, 465 (1981).
5. R. C. Ackerberg, *J. Fluid Mech.* 70, 333 (1975).
6. J.-M. Vanden-Broeck, *Phys. Fluids* (in press).

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